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# Epistemological Beliefs Regarding Mathematics Curriculum and Students' Academic Achievement

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## ABSTRACT

Epistemological beliefs about Mathematics are associated with students' cognitive performances and effective responses. This study aims to investigate the epistemological beliefs of students about the Mathematics curriculum and their relationship with academic achievement. In this study, a quantitative research approach through the correlational method was used. 600 students from 20 public secondary schools of the Lahore District were selected through two-stage random sampling. Conceptions of Mathematics Inventory were adapted after ensuring its validity and reliability in our local context. The reliability of the inventory was $\alpha = 0.917$. It was found that secondary school students had sophisticated epistemological beliefs about Mathematics curriculum beliefs about formulas; which are significant in Mathematics as they represent useful ideas and are based on the certainty of knowledge. Moreover, students' epistemological beliefs were positively correlated with their Mathematics academic achievements. It is recommended that teachers may try to recognize the Mathematics epistemological beliefs of students as they commence a new session of classes and take care of appropriate instructions that enhance students' Mathematics epistemological beliefs.

***Keywords:*** *Academic Achievements, Epistemological Beliefs, Mathematics, Secondary School Students*

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## INTRODUCTION

During the last decade, researchers are pursued to identify the connection between students' epistemological beliefs, their mathematical behavior, and performance. Because epistemological beliefs may impact on students' intellectual procedures of reasoning, originations of learning, and academic achievements (Hashmi, Khalid & Shoaib, 2019; Otting, Zwaal, Tempelaar, & Gijselaers, 2010; Phan, 2008; Sadi & Dağyar, 2015; Schommer, 1990, 1993). Ottmar and Landy (2017) found that students considered Mathematics as a repetition of symbols, focused on cramming rather than conceptualizing, and in the end, they failed in the examination. Szabo and Andrews (2018) found that many students believed that they could not solve a problem if they could not be solved within ten minutes. Moreover, they believe that obtaining the correct answers is the purpose of Mathematics.

Alexander, Murphy, and Sun (2017) identified the students' changes in Mathematics epistemological beliefs and task performance. They concluded that students who believed less certainty and authority of knowledge as the primary source of knowledge had better performance on the learning tasks. Moreover, students had the sophistication of beliefs about academic outcomes. Muis, Chevrier, and Singh (2018) seen that students' epistemological beliefs as dominatingly exact, i.e., students that evaluate mathematical beliefs on what is right or wrong execute more awful than students as overwhelmingly deductivist rational those evaluate rational beliefs through rationale. In any case, the analyst additionally uncovered that students that grasp a blend of deductivist and empiricist views didn't execute more terribly as compare to those with a prevalent deductivist view. Thus, students may have epistemological beliefs that come from two distinct epistemologies, for example, deductivist and empiricist point of view. Also, students with deductivist views had more sophisticated epistemological beliefs perform well than understudies with less sophisticated epistemological beliefs.

Students' mathematical epistemological belief about critical thinking is a significant segment of their learning experience, which influences their Mathematics learning activities and Mathematics achievement (Abedalaziz & Akmar, 2012). Nasser and Birenbaum (2005) said that there was an aberrant impact of epistemological beliefs on Mathematics achievement and gender was directly influenced theirs believes. Liu (2010) examined the unique view of two students in Mathematics (for example, imagining Mathematics as a procedure, including individual imagination) executed better on the identical calculus problems contrasted with a static instrumentalist view of two students in Mathematics as a lot of followed rules. So also, students' mathematical epistemological beliefs scores predicted their mathematical performance (Schommer-Aikins, Unruh, & Morphew, 2015).

Students' learning belief was associated with their academic progress. Moreover, mathematical performance is indirectly affected by general epistemological beliefs as

well as directly affected by domain-specific epistemological beliefs (Arslantaş, 2015, Rastegar, Jahromi, Haghighi, & Akbari, 2010; Schommer-Aikins & Duell, 2013). Köller (2001) provided evidence that constructive conception, certain knowledge, simple knowledge, and implication of Mathematics are the noteworthy indicators of high school students' mathematics achievement. Viholainen, Asikainen, and Hirvonen (2014) reported that students' beliefs about Mathematics as a static system and the formalism-related orientation. Moreover, The Mathematics curriculum prescribed for secondary classes has a wide range of concepts that must be learned and mastered by Pakistani students. It contains specialized knowledge that needs a certain frame of mind (i.e. Analytical and logical thinking). Its structure, operations, and processes provide students with a framework and tools for reasoning and expressing ideas clearly (Government of Pakistan, 2006). Thus, the present study was designed to measure secondary school students' EB about the Mathematics curriculum and explores their associations with students' academic achievement. This study also attempted to discriminate the students' epistemological beliefs based on gender. Consequently, the study may help to fill the gap in the existing literature.

## Research Gap

Epistemological beliefs about Mathematics are associated with students' cognitive performances and effective responses (Cam, Sulun, Topcu, & Guven, 2015). Teachers need to know the students' epistemological beliefs regarding the Mathematics curriculum (Ojose, 2015). In other words, determining epistemological beliefs supports teachers to interpret the learners' background and views regarding the subject that shapes their pedagogy.

Pakistan is far behind the developed countries, especially in the field of Mathematics and Science. Subsequently, most of the responsibilities laid on curriculum developers and teachers' methodology used to deliver knowledge. Consequently, it is required to explore students' epistemological beliefs regarding the Mathematics curriculum before developing the subject matter. Literature shows that few studies are conducted to examine students' epistemological beliefs about the Mathematics curriculum and their relationship with academic performance at a different level of education, especially in the Pakistani context. Researchers found the connection between students' Mathematics epistemological beliefs with their dimensions and academic achievement remain unanswered. Therefore, the purpose of this research was to determine students' epistemological beliefs regarding the Mathematics curriculum and the relationship between academic achievement at the secondary level.

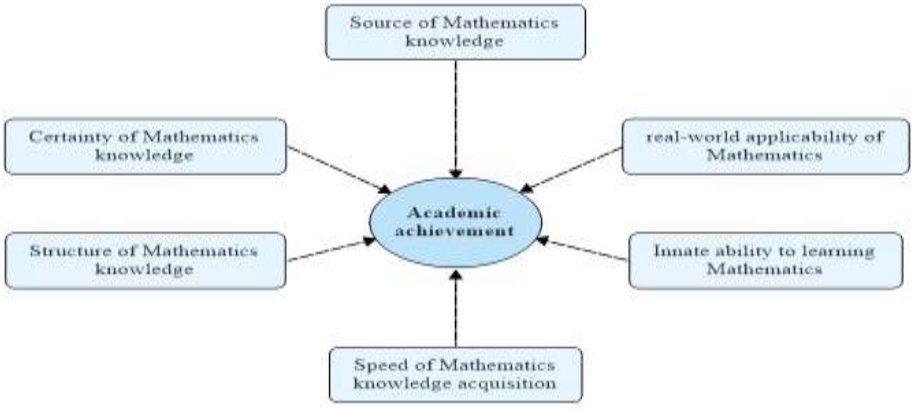


*Figure 1*: Conceptual Framework

## Research Objectives

1. Identify the students' epistemological beliefs about the Mathematics curriculum at the secondary level.
2. Distinguish students' epistemological beliefs of Mathematics curriculum based on gender.
3. Investigate the relationship in epistemological beliefs of students regarding Mathematics curriculum and academic achievement.

## Research Question/Hypotheses

1. What are the epistemological beliefs of students about the Mathematics curriculum at the secondary level?
2. There is no gender-wise significant difference in students' epistemological beliefs of the Mathematics curriculum.
3. There is no significant relationship between the epistemological beliefs of students regarding the Mathematics curriculum and their academic achievement.

## Theoretical Framework

The study depends on the theory of individual constructivism as epistemological beliefs have been connected with it from Perry (1970) and Piaget's leading work (1950; referred to in Colby, 2007). Piaget (1954) said that children modify either by assimilating new learning into what they already experience or accommodating their ideas to integrate new information. Guidance reliant on this theory and frequently attempted to incite

disequilibrium or disharmony, which is planned to make conceptual conflict and a short time later to help students to determine that contention (Gess-Newsome et al., 2016). As indicated by educational psychologists, epistemological beliefs are the individual's beliefs concerning the idea of the information that may incorporate beliefs for example obtaining assurance, source, and structure of information (Schommer-Aikins & Duell, 2013). Epistemological beliefs are frequently deciphered in Mathematics education research. As persons understanding and feelings that shape their concepts and participates in mathematical behavior. They set up a psychological setting for understanding and do Mathematics that might be exhibited as the most significant psychological construct for teacher education (Cook & Garneau, 2017). Epistemological beliefs are a significant piece of learning, thinking, data preparation, critical thinking, and achievement (Colby, 2007). Additionally, epistemological beliefs are related to students' performance (Cam et al., 2015).

## Literature Review

Previous researches showed that students' epistemological beliefs are associated with their mathematical behavior and performance (Alexander et al., 2017; Greene, Cartiff, & Duke, 2018; Hiebert & Grouws, 2007; Muis, 2004; Schommer-Aikins et al, 2015; Szabo & Andrews, 2018). Students' learning and views regarding the nature of knowledge have been investigated with the idea that they are part of the underlying mechanism of metacognition (Hashmi et al., 2019). They are more expected to suppose that knowledge is simple, certain, quick, and innate. Their conceptions of what counts as Mathematics are not only affected by their mathematical experiences at school but also by their personal out-of-school experiences (Schommer, 1993; Schommer-Aikins et al, 2015). Additionally, Sun, Kim, and Kang (2015) described that students enrolled in the graduate program were less likely than those who enrolled in the undergraduate program to have beliefs in certain and simple knowledge. In another study, Hui and Phang (2015) concluded that the epistemological beliefs of students became stronger (sophisticated) as they proceeded in their levels of education.

Epistemological beliefs are solid indicators of students' cognitive performances and successful reactions. Subsequently, their consequences for students' inclusion in learning, and the instructional methods pointed toward establishing an engaged learning climate (Magolda, 2014; Schommer, 1993). Similarly, Muis et al. (2018) expressed that students' epistemological viewpoints relate to their learning in different manners, such as impacting thinking and judgment. Investigators have generated additional proof regarding the non-availing and explicit nature of learners' mathematics-related epistemological beliefs (Buehl et al., 2002; Muis et al., 2018). Likewise, the accompanying patterns have arisen, for example, contributing to the student's overall beliefs just as domain-specific epistemological beliefs. Picker and Berry (2000)

considered that the generalized view of mathematicians as being male, very intelligent, obsessed with Mathematics, socially uncouth, and unanimously by elementary-level learners from different countries. Ottmar and Landy (2017) concluded that students that are failing in the assessment frequently see Mathematics as rote manipulation of symbols, concentration on remembrance as opposed to conceptualization.

Alexander et al. (2017) distinguished domain-specific epistemological beliefs of students and analyze contrasts in beliefs, inspiration, and task performance. They said that students who accepted less in the detachment and certainty of information, just as in power as the essential source of information were found to have more elevated levels of inspiration and better achievement. Aguilar et al. (2016) portrayed the views of high-achieving students that confirmed a similar generalized picture of mathematicians. Muis (2004) saw that students' epistemological beliefs as predominantly empirical, i.e., students that evaluate numerical truth on what aspects are wrong or right execute more awful than those as predominantly deductivist rational who evaluate numerical beliefs via logic. Notwithstanding, the investigator additionally uncovered that students that grasp a blend of deductivist and empiricist views didn't execute more awful as compared to those with a paramount deductivist view. Subsequently, learners may have epistemological beliefs that originate from two particular epistemologies for example the deductivist and empiricist viewpoint. Also, students with more advanced sophisticated epistemological beliefs (deductivist views) consistently perform in a way that is better than students with less sophisticated epistemological beliefs. Nasser and Birenbaum (2005) recognized that epistemological beliefs indirectly affected Mathematics achievement, attitudes, and Mathematics anxiety. Though it directly affected self-efficacy beliefs. To summarize, the accessible studies indicated that epistemological beliefs are commonly identified with pupils learning results: more powerful, fallibilist beliefs are normally connected with better educational results, a learner's hybrid set of epistemological beliefs encompassing both naive and sophisticated perspectives.

## Research Design

The study adopted the positivism paradigm. According to Creswell and Clark (2017), positivism is the best philosophy when statistical tools are used to collect and analyze data on a phenomenon in a scientific way. It is also used for testing objective theories by examining the relationship among variables. Correspondingly, a correlational survey method was applied to collect data and to investigate the secondary school students' Mathematics curriculum's epistemological beliefs and their relationship with academic achievement.

## Population and Sample

The population comprised of 46146 secondary school students who were studying in grade X (session 2017-19) in the 334 Public sector high schools of District Lahore. To

draw the sample of 600 students from 20 schools, two-stage stratified random sampling was used. According to Cochran (2007), a sample of 657 is required at a 99% confidence interval for the population of 46146. At the first stage, 20 public secondary schools (10 boys and 10 girls) were selected via disproportionate stratified random sampling technique and in the second stage, a simple random sampling method (without replacement method) was used for selecting the average 30 students of grade X (science section) from each chosen school.

## Research Instrument

The researchers used a questionnaire as the research instrument for the study. The questionnaire consists of two parts, i.e. demographics part (school name, gender, and marks of Mathematics subject obtained by $9^{th}$-grade examination) and the Conceptions of Mathematics Inventory (CMI was adapted with permission) which was developed by Grouws et al. (1996). CMI was a useful, context-free, valid, and reliable inventory for measuring students' Mathematics beliefs at the secondary level (Colby, 2007; Kennedy, 2009; Ilhan & Cetin, 2013; Slavin et al., 2009; Yang, 2013). CMI was adapted for the following reasons: i) developed to assess students' Mathematics epistemological beliefs; ii) the respondents of the studies also belonged to the secondary level. CMI consisted of seven dimensions with eight statements in each i.e. composition, structure, and status of Mathematical knowledge; doing, validating the idea, learning, and usefulness of Mathematics. It consisted of 56 items based on an agreement five-point Likert-type scale.

The validity of CMI was ensured by the three Pakistani experts in Mathematics education. They remarked on the ease of use, clarity, and revised some statements dimension wise. Similarly, CMI had low reliability ($\alpha$ = 0.306 to 0.619) that was not acceptable. Correspondingly, in the light of preliminary analysis, all items were redesigned under six dimensions of epistemological beliefs (Briley et al., 2009; Grouws et al., 1996; Woolfolk, 2016). The improved CMI was again validated through four experts in Mathematics education and assessment. After the validation process of CMI, Confirmatory Factor Analysis (CFA) was applied through smart PLS-3 and checked the internal consistency with the help of Cronbach's alpha coefficient. Assessment of cross-loadings, AVE, reliability, composite reliability, Fornell and Larcker criteria, and the correlations between dimensions or factors of Independent variables are enough for the results of the CFA (Lodhi, 2016; Wong, 2013).

The values of Cronbach's alpha and composite reliability of each dimension were more than 0.70 which were an acceptable level of constructing reliability. Cronbach's alpha value for Source of Mathematics knowledge (SMK); Certainty of Mathematics Knowledge (CMK); Structure of Mathematics Knowledge (STMK); Speed of Mathematics Knowledge Acquisition (SMKA); Innate Ability to Learning Mathematics

(IALM); and Real-World Applicability of Mathematics (RAM) were 0.722; 0.789; 0.746; 0.789; 0.799; and 0.818 respectively. Correspondingly, dimension-wise factor loading values were above 0.50 and significant, which were good signs indicators of the existence of reliability.

Table 1

*Fornell & Larcker criterion and Average Variance Extracted*

| CMI | AVE | CMK | IALM | RAM | SMK | SKA | STK |
|---|---|---|---|---|---|---|---|
| Certainty of Mathematics Knowledge | 0.568 | 0.754 | | | | | |
| Innate Ability to Learning Mathematics | 0.584 | 0.637 | 0.764 | | | | |
| Real-World Applicability of Mathematics | 0.595 | 0.820 | 0.709 | 0.871 | | | |
| Source of Mathematics knowledge | 0.584 | 0.724 | 0.714 | 0.798 | 0.864 | | |
| Speed of Mathematics Knowledge Acquisition | 0.570 | 0.768 | 0.719 | 0.730 | 0.729 | 0.774 | |
| Structure of Mathematics Knowledge | 0.571 | 0.733 | 0.831 | 0.724 | 0.716 | 0.755 | 0.866 |

Table 1 displays the values of Average Variance Extracted (AVE). These values show the existence of convergent validity as these values were greater than 0.5 of the cutoff point (Hair, Black, Babin, & Anderson, 2013). The discriminant validity of six dimensions of CMI has been checked from Fornell and Larcker (1981) criterion. During CFA three statements with less than 0.5-factor loading (λ) was excluded to enhance reliability. Finally, CMI had $\alpha = 0.917$ reliability with 53 statements.

## Data Analysis

The data were analyzed through descriptive and inferential statistics.

## Results of Study

RQ#1: What are the epistemological beliefs of students about the Mathematics curriculum at the secondary level?

Table 2

*Descriptive Statistics of CMI Dimensions*

| CMI | *M* | *SD* |
|---|---|---|
| Source of Mathematics Knowledge | 3.16 | 0.611 |
| Certainty of Mathematics Knowledge | 3.33 | 0.573 |

| | | |
|---|---|---|
| Structure of Mathematics Knowledge | 3.24 | 0.438 |
| Speed of Mathematics Knowledge Acquisition | 3.11 | 0.935 |
| Innate Ability to Learning Mathematics | 3.22 | 0.697 |
| Real-World Applicability of Mathematics | 3.07 | 0.535 |
| Mathematics Epistemological beliefs overall | 3.15 | 0.528 |

The table 2 demonstrates that in the certainty of Mathematics knowledge, participants had a strong (sophisticated) belief as $M$ = 3.33 and $SD$ = 0.573 than other Mathematics curriculum's epistemological beliefs. On the other hand, participants had weaknesses (naively) beliefs in real-world applicability of Mathematics as $M$ = 3.07 and $SD$ = 0.535. In addition, a slight difference was found in respondents' epistemological beliefs about innate ability and structure of mathematical knowledge as $M$ = 3.22 and 3.24 respectively. It is also found that students had a better belief about the speed of Mathematics knowledge acquisition ($M$ = 3.11) and source of Mathematics knowledge ($M$ = 3.16) in the comparison of the remaining dimensions.

$H_{o1}$: There is no gender-wise significant difference in students' epistemological beliefs of the Mathematics curriculum.

Table 3 *Independent Samples t-test for Students' Epistemological Beliefs about Mathematics curriculum*

| CMI | Boys (300) | | Girls (300) | | *df* | *t* | *p* |
|---|---|---|---|---|---|---|---|
| | *M* | *SD* | *M* | *SD* | | | |
| Source of Mathematics knowledge | 2.96 | 0.544 | 3.28 | 0.535 | 598 | -7.269 | 0.000 |
| Certainty of Mathematics Knowledge | 3.03 | 0.520 | 3.56 | 0.463 | 590.21 | -13.170 | 0.000 |
| Structure of Mathematics Knowledge | 3.17 | 0.432 | 3.35 | 0.376 | 586.73 | -5.333 | 0.000 |
| Speed of Mathematics Knowledge Acquisition | 2.69 | 0.885 | 3.28 | 0.852 | 598 | -8.270 | 0.000 |
| Innate Ability to Learning Mathematics | 2.81 | 0.571 | 3.38 | 0.581 | 598 | -12.063 | 0.000 |
| Real-World Applicability of Mathematics | 2.92 | 0.515 | 3.12 | 0.465 | 598 | -5.080 | 0.000 |
| Mathematics Epistemological beliefs overall | 2.94 | 0.353 | 3.35 | 0.355 | 598 | -14.089 | 0.000 |

Independent samples t-test was used to differentiate the epistemological beliefs of students about Mathematics curriculum based on gender. The result shows significant difference in overall epistemological beliefs of boy and girl students regarding Mathematics curriculum as the mean score of boy participants $M$=2.94, $SD$=0.353 < $M$=3.35, $SD$=0.355 girl participants as $t$ (598) = -14.089, p < 0.001, rejected the null hypothesis at two tailed). Hence, it is determined that girl students had strong (sophisticated) epistemological beliefs about Mathematics curriculum as compared to boy students. Similarly, significant difference is found in gender-based six dimensions of Mathematics curriculum's epistemological beliefs: such as source of Mathematics knowledge; certainty of Mathematics knowledge; structure of Mathematics knowledge; speed of Mathematics knowledge acquisition; innate ability to learning Mathematics; and real-world applicability of Mathematics) as p-value<0.05 i.e. $t$ (598) = -7.269, $p$ <0.001; $t$ (590.219) = -13.170, $p$ < 0.001; $t$ (586.733) = -5.333, $p$ < 0.001; $t$ (598) = -8.270, $p$ < 0.001; $t$ (598) = -12.063, $p$ < 0.001; and $t$ (598) = -5.080, $p$ <0.001 respectively.

$H_{o2}$: There is no significant relationship between the epistemological beliefs of students regarding the Mathematics curriculum and their academic achievement.

Table 4 *Correlation Matrix between in Epistemological Beliefs* of *Students about Mathematics Curriculum and their Academic Achievement*

| | SMK | CMK | STMK | SMKA | IALM | RAM | MEB | AA |
|---|---|---|---|---|---|---|---|---|
| Source of Mathematics knowledge | 1 | | | | | | | |
| Certainty of Mathematics Knowledge | .660** | 1 | | | | | | |
| Structure of Mathematics Knowledge | .600** | .658** | 1 | | | | | |
| Speed of Mathematics Knowledge Acquisition | .580** | .565** | .505** | 1 | | | | |
| Innate Ability to Learning Mathematics | .688** | .688** | .632** | .564** | 1 | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Real-World Applicability of Mathematics | .461** | .443** | .439** | .382** | .479** | 1 | | |
| Mathematics Epistemological beliefs overall | .835** | .868** | .819** | .699** | .872** | .645** | 1 | |
| Academic Achievement | .069** | .061* | .082** | .093** | .084** | .077** | .094** | 1 |

*Note:* **. Correlation is significant at the 0.01 level (2-tailed).

Correlation analysis was applied to scrutinize the relationship between students' academic achievement and their epistemological beliefs about the Mathematics curriculum. The results demonstrate all the epistemological beliefs dimensions of the Mathematics curriculum are significantly correlated with each other. Likewise, it is also revealed a significant relationship in students' academic achievement and all epistemological beliefs dimensions about Mathematics curriculum (source of Mathematics knowledge; the certainty of Mathematics knowledge; the structure of Mathematics knowledge; the speed of Mathematics knowledge acquisition; innate ability to learning Mathematics; and real-world applicability of Mathematics). Therefore, the null hypothesis is rejected.

According to Gravetter and Wallnau (2016) stated that if r = (+/-) 0 to 0.3, it expresses small and weak relationship in two variables. Correspondingly, the table shows that there was a positively weak relationship in all dimensions as source of Mathematics knowledge ($r$ = 0.069; $p$ = 0.008); certainty of Mathematics knowledge ($r$ = 0.061, $p$ = 0.019); structure of Mathematics knowledge ($r$ = 0.082, $p$ = 0.001); speed of Mathematics knowledge acquisition ($r$ = 0.093, $p < 0.001$); innate ability to learning Mathematics ($r$ = 0.084, $p$ = 0.001); and real-world applicability of Mathematics ($r$ = 0.77, $p$ = 0.003) as well as overall epistemological beliefs of students about Mathematics curriculum with their academic achievement ($r$ = 0.094, $p < 0.001$).

## Discussion

The analysis disclosed that students had certain beliefs about Mathematics which were based on Mathematical formulas. Similarly, previous researches accomplished that students have certain and quick beliefs about Mathematics (Hui & Phang, 2015; Schommer-Aikins et al, 2015; Sun et al., 2015; Szabo & Andrews, 2018). The finding showed that scores of students' epistemological beliefs about the Mathematics curriculum were significantly different due to gender. In the same way, Hakan and Munire (2012) concluded variation in students' epistemological beliefs based on gender.

Some studies revealed a significant cognitive difference in favor of girl students based on gender (Colby, 2007; ÖNEN, 2011; Marzooghi, Fouladchang, & Shemshiri, 2008; Schommer-Aikins & Easter, 2006). It was determined that girl students more strongly believed in learning depending on cognitive rather than effort compared to boy students (Arslantaş, 2015). The results showed that girls had more sophisticated beliefs than boys. Kurt (2009) concluded the same result. Yet, the outcome is contradicted with the result of the study which was done in China (Xiao, Yu, & Yan, 2009). Authors presumed no effect of gender orientation on students' Mathematics epistemological beliefs. The variation might exist because of context.

Consequently, it was found that students' academic achievement was positively related to their epistemological beliefs regarding the Mathematics curriculum. The outcomes are additionally following findings of researches conducted by Arslantaş (2015), Cano (2005), Colby (2007), and Tumkaya (2012). This result was affirmed by the researches, in which the presence of the positive connection between epistemological beliefs and students' academic achievements scores was determined. They reported that epistemological beliefs were the indicator of students' academic achievement (Garcia, 2005; Greene et al., 2018; Schommer, 1993; Schommer-Aikins & Easter, 2006).

## Conclusion

Epistemological beliefs have been associated with constructivism. In Mathematics, formulas are significant and represent useful ideas. It is concluded from the outcomes that students have sophisticated (strong) beliefs about the certainty of Mathematics knowledge belief. As they have strong beliefs on formula memorization. On the other hand, they believe that if they can't solve mathematics problems speedily, then devoting additional time won't help. This reflects their naive (weak) beliefs about the speed of Mathematics knowledge acquisition. Moreover, girl students have sophisticated epistemological beliefs as compare to boy students. This shows that boy and girl students have different Mathematics epistemological beliefs. Teachers might attempt to determine the students' Mathematics epistemological beliefs when they commence a new session of classes and take care of suitable instructions that enhance the boy students' Mathematics epistemological beliefs. Consequently, teachers may inspire boy students to have more strong Mathematics epistemological beliefs. The students' Mathematics epistemological beliefs might be discussed with them and their parents by parents-teachers meetings. So that parents might improve their children's beliefs at home.

The result of this study shows that students' academic achievement is positively weak correlated with their Mathematics epistemological beliefs overall and their dimensions. Further, the examination might be led to decide the impacts of the financial status on epistemological beliefs of students as the present study was limited to find the relationship of students' epistemological beliefs and academic achievement.

It is hard to determine individuals' epistemological beliefs from a solitary report by utilizing one instrument or conceptual framework. Theoretical models of epistemological beliefs proceed to advance, and researchers keep on creating techniques for surveying beliefs. On the off chance that another instrument or framework was utilized various outcomes might be found. Some researchers have recommended utilizing blends of frameworks to explore individuals' epistemological beliefs. Therefore, future researchers may evaluate individuals' epistemological beliefs about Mathematics with a mix of instruments (Colby, 2007; Schommer-Aikins & Easter, 2006).

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